

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2024

SECOND YEAR (BATCH 2022-24)

Date : 27/05/2024

Time : 11.00 am – 1.00 pm

MATHEMATICS

Paper : CC 19

Full Marks : 50

Continuum Mechanics II

[Use a separate Answer Book for each group]

Group : A

[All the symbols have their usual meaning]

Answer all the questions, maximum you can score is 25.

1. Obtain Beltrami-Michell stress compatibility equations. [5]
2. Write short notes on (i) irrotational waves (ii) equivoluminal waves. [2.5+2.5]
3. In case of equilibrium of elastic body under no body forces, prove that strain invariant and stress invariant are harmonic functions. [2+2]
4. What is Airy's Stress function and how it is related with stresses? [3]
5. Show that solution of fundamental boundary value problems in elastodynamics is unique. [3]
6. Write short notes on
(i) P waves, (ii) S waves. [2+2]
7. If M_A, M_B, M_C denotes bending moments of a uniformly loaded beam at three successive points of support A, B, C which are on the same horizontal line. Prove that
 $aM_A + 2(a+b)M_B + bM_C = w(a^3 + b^3)/4$ where w is the weight of the beam per unit length,
 $AB = a, BC = b$. [6]

Group : B

Answer all the questions, maximum you can score is 25.

8. Define acyclic and cyclic motion. [2.5]
9. Prove that acyclic irrotational motion of a liquid at rest at infinity, due to the prescribed motion of an immersed solid, is uniquely determined by the motion of the solid. [2.5]
10. State and prove Kelvin's circulation theorem. [5]
11. Find the velocity potential and lines of flow of a sphere through an infinite mass of a liquid at rest at infinity. [5]
12. The space between concentric spherical shells of radii a and b ($a > b$) is filled with an incompressible fluid of density ρ and the shells suddenly begin to move with velocities U, V in the same direction. Find the resultant impulsive pressure on the inner shell. Also find the kinetic energy of the liquid. [5]
13. Obtain the equation of energy for a viscous compressible fluid. [8]
14. What is three dimensional doublet? [2]

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[P.T.O.]

Analytic Number Theory

[All the symbols have their usual meaning]

Answer **all** the questions. Maximum you can score is 50.

1. Let S be the sequence of all integers n such that $\gcd(n, pq) = 1$, where p, q are odd distinct primes. What is the maximum distance between consecutive elements of S , S being written in increasing order. [5]
2. Find all $n \in \mathbb{N}$ such that $\phi(n) = \frac{n}{3}$. [5]
3. Show that 2 is a primitive root modulo 3^e for all $e \in \mathbb{N}$. [6]
4. Find all primes such that $-3 \in Q_p$. [7]
5. Prove that each prime $p \equiv 1 \pmod{4}$ is a sum of two squares. [7]
6. Let f be a multiplicative arithmetic function. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \in \mathbb{N}$. [7]
7. Prove that for all $X \geq 1$, we have
$$\sum_{n \leq X} d(n) = X \log X + (2\gamma - 1)X + O(\sqrt{X}),$$
 where γ is Euler's constant. [8]
8. Use Prime Number Theorem to prove that every interval $[a, b]$ with $0 < a < b$, contains a rational number of the form $\frac{p}{q}$, where p and q are primes. [6]
9. Calculate the highest exponent $k \in \mathbb{N}$ such that $10^k \mid 1000!$. [4]
10. Solve the congruence $x^2 \equiv 6 \pmod{5^3}$. [5]

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